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Knotlike cosmic strings in the early universe

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Abstract: In this paper, the knotlike cosmic strings in the Riemann-Cartan space-time of the early universe are discussed. It has been revealed that the cosmic strings can just originate from the zero points of the complex scalar quintessence field. In these strings we mainly study the knotlike configurations. Based on the integral of Chern-Simons 3-form a topological invariant for knotlike cosmic strings is constructed, and it is shown that this invariant is just the total sum of all the self-linking and linking numbers of the knots family. Furthermore, it is also pointed out that this invariant is preserved in the branch processes during the evolution of cosmic strings.

Keywords: Topological Field Theories, Physics of the Early Universe.

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1. Introduction

The recent measurements of the redshift of type Ia supernova (SN Ia) and the power spectrum of the cosmic microwave background (CMB) from BOOMERANG-98 and MAXIMA-1 respectively suggest the accelerating expansion and the flatness of the universe [1, 2]. These two observations lead to the conclusion that the universe has the critical density and consists of 1/3 of ordinary matter and 2/3 of dark energy with negative pressure. At the moment, a most-often considered candidate for dark energy is the *quintessence* [3]. In the present paper, we mainly consider the complex scalar quintessence field $\psi(x)$ on the Riemann-Cartan manifold ($\mathbf{U}^4$) of the early universe [4]. As a background field of the universe, $\psi(x)$ is a section of the complex line bundle, i.e., a section of the two-dimensional real vector bundle on $\mathbf{U}^4$:

$$\psi(x) = \phi^1(x) + i\phi^2(x)\,. \tag{1.1}$$

In this paper, in the following section 2, it will be shown that the cosmic string structures can just originate from the zero points of this complex scalar quintessence field $\psi(x)$.

In 1997, Faddeev and Niemi pointed out that for a string structure of finite energy, its length must be finite, which is possible if its core forms a knot. It is shown that in a realistic $(3+1)$-dimensional model there exist knotlike structures appearing as stable finite solitons [5]. Since then knotlike configurations as string structures of finite energy are paid close attention to in a variety of physical, chemical and biological scenarios, including the structure of elementary particles [6], the early universe cosmology [7, 8], the Bose-Einstein condensation [9], the polymer folding [10], and the DNA replication, transcription and recombination [11]. In the following section 3, we will emphasize on the knotlike cosmic strings and study the topological invariant for these knots.

This paper is arranged as follows. In section 2, we use the ϕ-mapping topological current theory [12, 13] to reveal that the cosmic strings can just originate from the zero points of quintessence field $\psi(x)$. In this section the Nielsen lagrangian and Nambu action of these strings are also simply discussed. In section 3, we mainly study the knotlike configurations in these string structures. Based on the integral of Chern-Simons 3-form we construct a topological invariant for the knotlike strings. It is shown that this invariant is just the total sum of all the self-linking and linking numbers of the knots family. In section 4, moreover, the conservation of this topological invariant in the branch processes (i.e. the splitting, mergence and intersection) during the evolution of knotlike cosmic strings is simply discussed.

2. The cosmic strings

In this section, it is pointed out that the cosmic strings can originate from the zero points of the complex scalar quintessence field $\psi(x)$. The Nielsen lagrangian and Nambu action of these strings are also simply discussed.

The U(1) gauge field tensor, i.e. the curvature of U(1) principal bundle on $\mathbf{U}^4$, is written as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{2.1}$$

where $\mu = 0, 1, 2, 3$ denotes the base manifold, and A_μ is the U(1) gauge potential, i.e. the connection of U(1) principal bundle on $\mathbf{U}^4$. Define A_μ possesses the inner structure as

$$A_\mu = \frac{\alpha}{2\pi}\frac{1}{2i\psi^*\psi}(\psi^*\partial_\mu\psi - \partial_\mu\psi^*\psi) \,, \tag{2.2}$$

where $\alpha = \sqrt{\hbar G/c^3}$ is a constant. It can be proved that under the U(1) gauge transformation of $\psi(x)$: $\psi' = e^{i\lambda}\psi$, A_μ satisfies the U(1) transformation relation:

$$A'_\mu = A_\mu + i\frac{\alpha}{2\pi}\partial_\mu\lambda \,, \tag{2.3}$$

where $\lambda \in \mathbb{R}$ is the transformation parameter. (In (2.2) the *RHS* has taken the form of velocity field in quantum mechanics. In superconductivity theory, the form of (2.2) actually corresponds to the London relation [14]).

Introducing the two-dimensional unit vector n^a from $\phi^{1,2}$:

$$n^a = \frac{\phi^a}{\|\phi\|}, \qquad (a = 1, 2;\ \|\phi\|^2 = \phi^a\phi^a = \psi^*\psi) \tag{2.4}$$

the expression (2.2) can be written as

$$A_\mu = \frac{\alpha}{2\pi}\epsilon_{ab}n^a\partial_\mu n^b \,, \tag{2.5}$$

and then the field tensor $F_{\mu\nu}$ is just

$$F_{\mu\nu} = \frac{\alpha}{2\pi}2\epsilon_{ab}\partial_\mu n^a\partial_\nu n^b \,. \tag{2.6}$$

According to the ϕ-mapping topological current theory [12], in Riemann-Cartan space-time the topological tensor current is defined as

$$j^{\mu\nu} = \frac{1}{2}\frac{1}{\sqrt{g}}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho} \qquad (g = \det\{g_{\mu\nu}\}) \,. \tag{2.7}$$

It is seen that (2.7) satisfies

$$j^{\mu\nu} = -j^{\nu\mu}\,, \qquad \nabla_\mu j^{\mu\nu} = \frac{1}{\sqrt{g}}\partial_\mu(\sqrt{g}j^{\mu\nu}) = 0\,, \tag{2.8}$$

hence $j^{\mu\nu}$ is anti-symmetric and is an identically conserved current.

Using $\partial_\mu n^a = \frac{\partial_\mu \phi^a}{\|\phi\|} + \phi^a \partial_\mu \frac{1}{\|\phi\|}$ and the Green function relation in ϕ-space: $\partial_a \partial_a \ln \|\phi\|$ $= 2\pi\delta^2(\vec{\phi})$(where $\partial_a = \partial/\partial\phi^a$), it can be proved that $j^{\mu\nu}$ may be expressed in a δ-function form:

$$j^{\mu\nu} = \frac{\alpha}{\sqrt{g}}\delta^2(\vec{\phi})D^{\mu\nu}\left(\frac{\phi}{x}\right), \tag{2.9}$$

where $D^{\mu\nu}(\phi/x) = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\epsilon_{ab}\partial_\rho\phi^a\partial_\lambda\phi^b$; while the spatial component of $j^{\mu\nu}$ is just

$$j^i = j^{0i} = \frac{1}{2\sqrt{g}}\epsilon^{ijk}F_{jk} = \frac{\alpha}{\sqrt{g}}\delta^2(\vec{\phi})D^i\left(\frac{\phi}{x}\right), \qquad (i,j,k = 1,2,3)\,, \tag{2.10}$$

where $D^i(\phi/x) = \frac{1}{2}\epsilon^{ijk}\epsilon_{ab}\partial_j\phi^a\partial_k\phi^b$ is the jacobian vector.

Obviously the expression (2.9) provides an important conclusion:

$$j^{\mu\nu}\begin{cases} = 0\,, & \text{iff} \quad \vec{\phi} \neq 0\,; \\ \neq 0\,, & \text{iff} \quad \vec{\phi} = 0\,, \end{cases} \tag{2.11}$$

so it is necessary to study the zero points of $\vec{\phi}$ to determine the non-zero solutions of $j^{\mu\nu}$. The implicit function theory shows [15] that under the regular condition

$$D^{\mu\nu}\left(\frac{\phi}{x}\right) \neq 0\,, \tag{2.12}$$

the general solutions of

$$\phi^1(x^0 = t,\ x^1,\ x^2,\ x^3) = 0\,, \qquad \phi^2(x^0 = t,\ x^1,\ x^2,\ x^3) = 0 \tag{2.13}$$

can be expressed as

$$x^\mu = x^\mu_k(u^1,\ u^2)\,, \qquad (\mu = 0,1,2,3) \tag{2.14}$$

which represents N two-dimensional singular submanifolds P_k $(k = 1,2,\ldots,N)$ with intrinsic coordinates u^1 and u^2. In this paper, in particular, u^1 and u^2 respectively take the space-like string parameter s and time-like evolution parameter t (i.e. $u^1 = s,\ u^2 = t$), then the N submanifolds P_k's are just the world surfaces of a family of N moving isolated singular strings L_k's. These singular string solutions are just the cosmic strings.

Next we should expand $j^{\mu\nu}$ onto these N singular submanifolds P_k's. First, it can be proved that in the four-dimensional space-time there exists a two-dimensional submanifold M which is transversal to every P_k at the section point p_k:

$$g_{\mu\nu}\frac{\partial x^\mu}{\partial u^I}\frac{\partial x^\nu}{\partial v^C}\bigg|_{p_k} = 0\,, \qquad (I = 1,2;\ C = 1,2)\,, \tag{2.15}$$

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where v^1 and v^2 are the intrinsic coordinates of M. Then on M one can prove that [16, 12]

$$\delta^2(\vec{\phi}) = \sum_{k=1}^{N} \beta_k \eta_k \delta^2(\vec{v} - \vec{v}(p_k)) \,, \tag{2.16}$$

where the positive integer β_k is the Hopf index and $\eta_k = \pm 1$ the Brouwer degree of ϕ-mapping, and $W_k = \beta_k \eta_k$ is just the winding number of string L_k. Second, since every p_k is related to a singular submanifold P_k, the above two-dimensional δ-function of singular point (i.e. $\delta^2(\vec{v} - \vec{v}(p_k))$) must be expanded to the δ-function on singular submanifold P_k (i.e. $\delta(P_k)$). Meanwhile in δ-function theory it has been given that [16, 12]

$$\delta(P_k) = \int_{P_k} \delta^4(x^\mu - x_k^\mu(u)) \sqrt{g_u} d^2u \,, \tag{2.17}$$

where g_u is the determinant of the metric g_{IJ} of P_k: $g_u = \det(g_{IJ})$ $(I, J = 1, 2;\ g_{IJ} = g_{\mu\nu} \frac{\partial x^\mu}{\partial u^I} \frac{\partial x^\nu}{\partial u^J})$. So, third, from (2.17) and (2.16), we have

$$\delta^2(\vec{\phi}) = \sum_{k=1}^{N} \beta_k \eta_k \int_{P_k} \delta^4(x^\mu - x_k^\mu(u)) \sqrt{g_u} d^2u \,, \tag{2.18}$$

and therefore $j^{\mu\nu}$ is expanded onto N singular submanifolds P_k:

$$j^{\mu\nu} = \frac{\alpha}{\sqrt{g}} D^{\mu\nu} \left(\frac{\phi}{x} \right) \sum_{k=1}^{N} \beta_k \eta_k \int_{P_k} \delta^4(x^\mu - x_k^\mu(u)) \sqrt{g_u} d^2u \,. \tag{2.19}$$

In (2.19) the spatial components of $j^{\mu\nu}$ is

$$j^i = j^{0i} = \frac{1}{2\sqrt{g}} \epsilon^{ijk} F_{jk} = \frac{\alpha}{\sqrt{g}} \sum_{k=1}^{N} W_k \int_{L_k} \frac{dx^i}{ds} \delta^3(\vec{x} - \vec{x}_k(s)) ds \,, \qquad (W_k = \beta_k \eta_k) \,, \tag{2.20}$$

where $\frac{dx^i}{ds} = \frac{D^i(\phi/x)}{D(\phi/u)}$. Hence the topological charge of string L_k is just

$$Q_k = \frac{1}{\alpha} \int_{\Sigma_k} j^i \sqrt{g} d\sigma_i = W_k \,, \tag{2.21}$$

where Σ_k is the two-dimensional spatial surface element perpendicular to L_k.

In the end of this section we would also simply discuss the Nambu action of these N cosmic strings L_k's. Define the lagrangian of these strings as

$$L = \frac{1}{\alpha} \sqrt{\frac{1}{2} g_{\mu\lambda} g_{\nu\rho} j^{\mu\nu} j^{\lambda\rho}} \,, \tag{2.22}$$

which is just the generalization of Nielsen's lagrangian [17]. Then from the above deduction (2.9) one can prove

$$L = \frac{1}{\sqrt{g}} \delta(\vec{\phi}) \,. \tag{2.23}$$

So the action is just

$$S = \int_{\mathbf{U}^4} L \sqrt{g} d^4x = \int_{\mathbf{U}^4} \delta(\vec{\phi}) d^4x \,. \tag{2.24}$$

Substituting (2.18) into (2.24):

$$S = \int_{\mathbf{U}^4} \sum_{k=1}^{N} \beta_k \eta_k \int_{P_k} \delta^4(x - x_k(u)) \sqrt{g_u} d^2u d^4x$$
$$= \sum_{k=1}^{N} \beta_k \eta_k \int_{P_k} \sqrt{g_u} d^2u \,, \tag{2.25}$$

we arrive at an important result

$$S = \sum_{k=1}^{N} \beta_k \eta_k S_k \,, \tag{2.26}$$

where $S_k = \int_{P_k} \sqrt{g_u} d^2u$ is the area of singular submanifold P_k. Therefore the expression (2.26) is just the Nambu action of the N cosmic strings [17, 7], which is the basis of the further work on cosmic string theory.

Furthermore, from the principle of least action, we can also obtain the evolution equation for these N strings as [12]

$$\frac{1}{\sqrt{g_u}} \frac{\partial}{\partial u^I} \left(\sqrt{g_u} g^{IJ} \frac{\partial x^\mu}{\partial u^J} \right) + g^{IJ} \Gamma^\mu_{\nu\lambda} \frac{\partial x^\nu}{\partial u^I} \frac{\partial x^\lambda}{\partial u^J} = 0 \,, \qquad (I, J = 1, 2) \,. \tag{2.27}$$

Finally, it should be addressed that in the above text the regular condition (2.12) has been used; when this condition fails, the branch processes during the evolution of cosmic strings will occur. This will be detailed in section 4.

3. Topological invariant for knotlike cosmic strings

In this section, based on the integral of Chern-Simons 3-form we mainly study the topological invariant for the knotlike cosmic strings.

In order to construct a topological invariant in the space-time, one must pick an integral expression which does not require any choice of metric $g_{\mu\nu}$. Precisely in three-dimensional space there is a reasonable choice, namely, the integral of the Chern-Simons 3-form [18, 19]:

$$Q = \left(\frac{2\pi}{\alpha} \right)^2 \frac{1}{4\pi} \int_\Omega \epsilon^{ijk} A_i F_{jk} d^3x \,, \tag{3.1}$$

where Ω is the spatial volume. Hereinafter we just study (3.1) to get the topological invariant for the knotlike cosmic strings.

Using the above (2.10) and (2.20), the expression (3.1) can be written as

$$Q = \frac{2\pi}{\alpha} \int_\Omega A_i \delta^2(\vec{\phi}) D^i \left(\frac{\phi}{x} \right) d^3x = \frac{2\pi}{\alpha} \sum_{k=1}^{N} W_k \int_{L_k} A_i dx^i. \tag{3.2}$$

It can be seen that when the N cosmic strings of (3.2) are N closed curves, i.e., a family of N knotlike strings γ_k $(k = 1, \ldots, N)$, (3.2) leads to

$$Q = \frac{2\pi}{\alpha} \sum_{k=1}^{N} W_k \oint_{\gamma_k} A_i dx^i \,. \tag{3.3}$$

This is a very important expression. Considering the U(1) gauge transformation of A_i in (2.3): $A_i' = A_i + i\frac{\alpha}{2\pi}\partial_i\lambda$, it can be seen that the $(i\frac{\alpha}{2\pi}\partial_i\lambda)$ term contributes nothing to the integral Q, hence the expression (3.3) is invariant under the gauge transformation. Therefore, from the fact that Q of (3.3) is independent of the choice of metric and is invariant under the U(1) gauge transformation, one can conclude that Q is a topological invariant for the knotlike cosmic strings. This can be used in the research of the topology of string structures in the early universe. At the same time, for the Chern-Simons integral itself, (3.3) provides a sufficiency condition for the Chern-Simons integral to be a topological invariant, which is another significance of expression (3.3).

In following we will show that Q is just the total sum of all the self-linking and linking numbers of the knotlike strings family. Using (2.10), the expression (3.3) can be reexpressed as

$$Q = \frac{2\pi}{\alpha}\sum_{k=1}^{N}\sum_{l=1}^{N} W_k W_l \oint_{\gamma_k}\oint_{\gamma_l} \partial_i A_j dx^i dy^j \,, \tag{3.4}$$

where $\vec{x}$ and $\vec{y}$ are two points respectively on knots γ_k and γ_l. Noticing that γ_k and γ_l can be the same one knot, or two different knots, we should write (3.4) in two parts ($k = l$ and $k \neq l$); furthermore the $k = l$ part includes both the $\vec{x} \neq \vec{y}$ and the $\vec{x} = \vec{y}$ cases. So totally Q should be written in three terms: ($k = l$; $\vec{x} \neq \vec{y}$), ($k = l$; $\vec{x} = \vec{y}$) and ($k \neq l$).

Define a three-dimensional unit vector

$$\vec{m} = \frac{\vec{y} - \vec{x}}{\|\vec{y} - \vec{x}\|}, \tag{3.5}$$

and another two-dimensional unit vector $\vec{e}$ on the $\vec{m}$ -formed-sphere S^2 :

$$\vec{e}\perp\vec{m}\,, \qquad \vec{e}\cdot\vec{e} = 1\,. \tag{3.6}$$

The vector field $\vec{e}$ is the section of two-dimensional real vector bundle, i.e., the section of complex line bundle

$$\chi = e^1 + ie^2\,, \tag{3.7}$$

where χ is the complex scalar field. Then, similarly as in section 2, one can give out the inner structure of A_i in terms of $\vec{e}$ (i.e. χ) as

$$A_i = \frac{\alpha}{2\pi}\epsilon_{ab}e^a\partial_i e^b\,, \qquad (a,\ b = 1,2;\ i = 1,2,3)\,. \tag{3.8}$$

Using (3.8) and the relation $2\epsilon_{ab}\partial_i e^a\partial_j e^b = \vec{m}\cdot(\partial_i\vec{m}\times\partial_j\vec{m})$, the three terms of Q can be expressed as

$$\begin{aligned} Q = 2\pi\Big[&\sum_{k=1\ (\vec{x}\neq\vec{y})}^{N} \frac{1}{4\pi}W_k^2\oint_{\gamma_k}\oint_{\gamma_k}\vec{m}^*(dS) + \frac{1}{2\pi}\sum_{k=1}^{N}W_k^2\oint_{\gamma_k}\epsilon_{ab}e^a\partial_i e^b dx^i + \\ &+\sum_{k,l=1\ (k\neq l)}^{N}\frac{1}{4\pi}W_kW_l\oint_{\gamma_k}\oint_{\gamma_l}\vec{m}^*(dS)\Big]\,. \end{aligned} \tag{3.9}$$

where $\vec{m}^*(dS) = \vec{m}\cdot(\partial_i\vec{m}\times\partial_j\vec{m})dx^i\wedge dy^j$ ($\vec{x}\neq\vec{y}$) denotes the pull-back of S^2 surface element.

Let us discuss these three terms in detail. First, the first term of (3.9) is just related to the writhing number $Wr(\gamma_k)$ of γ_k [20]:

$$Wr(\gamma_k) = \frac{1}{4\pi} \oint_{\gamma_k} \oint_{\gamma_k} \vec{m}^*(dS) \,. \tag{3.10}$$

For the second term of (3.9), since this is the $\vec{x} = \vec{y}$ term, one can prove that it is related to the twisting number $Tw(\gamma_k)$ of γ_k [20]

$$\frac{1}{2\pi} \oint_{\gamma_k} \epsilon_{ab} e^a \partial_i e^b dx^i = \frac{1}{2\pi} \oint_{\gamma_k} (\vec{T} \times \vec{V}) \cdot d\vec{V} = Tw(\gamma_k) \,, \tag{3.11}$$

where $\vec{T}$ is the unit tangent vector of knot γ_k at $\vec{x}$ [$\vec{m} = \vec{T}$ when $\vec{x} = \vec{y}$], and $\vec{V}$ is defined as $e^a = \epsilon^{ab} V^b$ ($\vec{V} \perp \vec{T}$, $\vec{e} = \vec{T} \times \vec{V}$). From the White formula [20]

$$SL(\gamma_k) = Wr(\gamma_k) + Tw(\gamma_k) \tag{3.12}$$

one see that the first and second terms of (3.9) just compose the self-linking numbers of knots.

Second, for the third term, one can prove

$$\frac{1}{4\pi} \oint_{\gamma_k} \oint_{\gamma_l} \vec{m}^*(dS) = \frac{1}{4\pi} \epsilon^{ijk} \oint_{\gamma_k} dx^i \oint_{\gamma_l} dy^j \frac{(x^k - y^k)}{\|\vec{x} - \vec{y}\|^3} = Lk(\gamma_k, \gamma_l) \;\; (k \neq l) \tag{3.13}$$

where $Lk(\gamma_k, \gamma_l)$ is the Gauss linking number between γ_k and γ_l[19, 21].

Therefore, third, from (3.10), (3.11), (3.12) and (3.13), we arrive at the important result:

$$Q = 2\pi \left[\sum_{k=1}^{N} W_k^2 SL(\gamma_k) + \sum_{k,l=1\ (k \neq l)}^{N} W_k W_l Lk(\gamma_k, \gamma_l) \right] . \tag{3.14}$$

This precise expression just reveals the relationship between Q and the self-linking and linking numbers of the knots family [19, 22, 23]. Since the self-linking and linking numbers are both the intrinsic invariant characteristic numbers of knots family in topology, expression (3.14) directly relates Q to the topology of the knots family itself, and therefore Q can be regarded as an important invariant required to describe the topology of knotlike cosmic strings in early universe. This is just the significance of the introduction and research of topological invariant Q.

4. Conservation of Q in the branch processes of knotlike cosmic strings

In our previous work [24] it has been pointed out that, during the evolution of cosmic strings, when the regular condition (2.12) fails, the branch processes (i.e. the splitting, mergence and intersection) will occur; and in these branch processes, the sum of the topological charges of all the final cosmic string(s) is equal to that of all the initial cosmic string(s) at the bifurcation point, namely:

(a) for the case that one string L split into two strings L_1 and L_2, we have $W_L = W_{L_1} + W_{L_2}$;

(*b*) the case that two strings L_1 and L_2 merge into one string L: $W_{L_1} + W_{L_2} = W_L$;

(*c*) the case that two strings L_1 and L_2 meet, and then depart as two other strings L_3 and L_4: $W_{L_1} + W_{L_2} = W_{L_3} + W_{L_4}$.

In following we will show that when the branch processes of knotlike strings occur, the topological invariant Q of (3.3) (i.e. (3.14)) is preserved:

(i) The splitting case. We will consider one knot γ split into two knots γ_1 and γ_2 which are of the same self-linking number as γ ($SL(\gamma) = SL(\gamma_1) = SL(\gamma_2)$), and then we will compare the two numbers Q_γ and $Q_{\gamma_1+\gamma_2}$ [where Q_γ is the contribution of γ to Q before splitting, and $Q_{\gamma_1+\gamma_2}$ is the total contribution of γ_1 and γ_2 to Q after splitting]. First, from the above text we have $W_\gamma = W_{\gamma_1} + W_{\gamma_2}$ in the splitting process. Second, on the one hand, in the neighborhood of bifurcation point $(\vec{x}^*,\ t^*)$, γ_1 and γ_2 are infinitesimally displaced from each other; on the other hand, for a knot γ its self-linking number $SL(\gamma)$ is defined as

$$SL(\gamma) = Lk(\gamma,\ \gamma_V)\,, \tag{4.1}$$

where γ_V is another knot obtained by infinitesimally displacing γ in the normal direction $\vec{V}$ [19]. Therefore

$$SL(\gamma) = SL(\gamma_1) = SL(\gamma_2) = Lk(\gamma_1,\ \gamma_2) = Lk(\gamma_2,\ \gamma_1)\,, \tag{4.2}$$

and

$$Lk(\gamma,\ \gamma'_k) = Lk(\gamma_1,\ \gamma'_k) = Lk(\gamma_2,\ \gamma'_k) \tag{4.3}$$

(where γ'_k denotes another arbitrary knot in the family ($\gamma'_k \neq \gamma,\ \gamma'_k \neq \gamma_{1,2}$)). Then, third, we can compare Q_γ and $Q_{\gamma_1+\gamma_2}$ as: before splitting, from (3.14) we have

$$Q_\gamma = 2\pi \left[W_\gamma^2 SL(\gamma) + \sum_{k=1\ (\gamma'_k \neq \gamma)}^{N} 2W_\gamma W_{\gamma'_k} Lk(\gamma, \gamma'_k) \right], \tag{4.4}$$

where $Lk(\gamma, \gamma'_k) = Lk(\gamma'_k, \gamma)$; after splitting,

$$\begin{aligned} Q_{\gamma_1+\gamma_2} = 2\pi \Big[& W_{\gamma_1}^2 SL(\gamma_1) + W_{\gamma_2}^2 SL(\gamma_2) + 2W_{\gamma_1} W_{\gamma_2} Lk(\gamma_1, \gamma_2) + \\ & + \sum_{k=1\ (\gamma'_k \neq \gamma_{1,2})}^{N} 2W_{\gamma_1} W_{\gamma'_k} Lk(\gamma_1, \gamma'_k) + \sum_{k=1\ (\gamma'_k \neq \gamma_{1,2})}^{N} 2W_{\gamma_2} W_{\gamma'_k} Lk(\gamma_2, \gamma'_k) \Big]\,. \end{aligned} \tag{4.5}$$

Comparing (4.4) and (4.5), we just have

$$Q_\gamma = Q_{\gamma_1+\gamma_2}\,. \tag{4.6}$$

This means that in the splitting process Q is preserved.

(ii) The mergence case. We consider two knots γ_1 and γ_2, which are of the same self-linking number, merge into one knot γ which is of the same self-linking number as γ_1 and γ_2. This is obviously the inverse process of the above splitting case, therefore we have

$$Q_{\gamma_1+\gamma_2} = Q_{\gamma}\,. \tag{4.7}$$

(iii) The intersection case. This case is related to the collision of two knots [8]. We consider two knots γ_1 and γ_2, which are of the same self-linking number, meet, and then depart as other two knots γ_3 and γ_4 which are of the same self-linking number as γ_1 and γ_2. This process can be identified to two sub-processes: γ_1 and γ_2 merge into one knot γ, and then γ split into γ_3 and γ_4. Thus from the above two cases (ii) and (i) we have

$$Q_{\gamma_1+\gamma_2} = Q_{\gamma_3+\gamma_4}\,. \tag{4.8}$$

Therefore we obtain the result that, in the branch processes during the evolution of knotlike cosmic strings (i.e., the splitting, mergence and intersection), the topological invariant Q is preserved.

5. Conclusion

In this paper, the complex scalar quintessence field on the Riemann-Cartan manifold of the early universe is considered. In section 2, it is revealed that from the U(1) field tensor $F_{\mu\nu}$ one can derive the cosmic string structures, which just originate from the zero points of quintessence field $\psi(x)$. In this section the Nielsen lagrangian and Nambu action of these strings are also simply discussed. In section 3, we emphasize on the knotlike configurations in these cosmic strings. Based on the integral of Chern-Simons 3-form, we construct a topological invariant Q for the knotlike strings. It is pointed out that Q is just the total sum of all the self-linking and linking numbers of the knots family. In section 4, it is shown that Q is preserved in the branch processes (i.e. the splitting, mergence and intersection) during the evolution of knotlike cosmic strings.

At last there are two points which should be addressed. First, in this paper we treat the cosmic strings as mathematical lines, i.e., the width of a string is zero. This description is obtained in the approximation that the radius of curvatures of a string is much larger than the width of the string [17]. The further research on the width of cores of strings will be detailed in our later papers. Second, in this paper the Nambu action of cosmic strings has been simply discussed. Furthermore, the research of the Higgs lagrangian (i.e. the Ginzburg-Landau free energy) of knots as well as the classification of knots will be detailed in our further work.

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